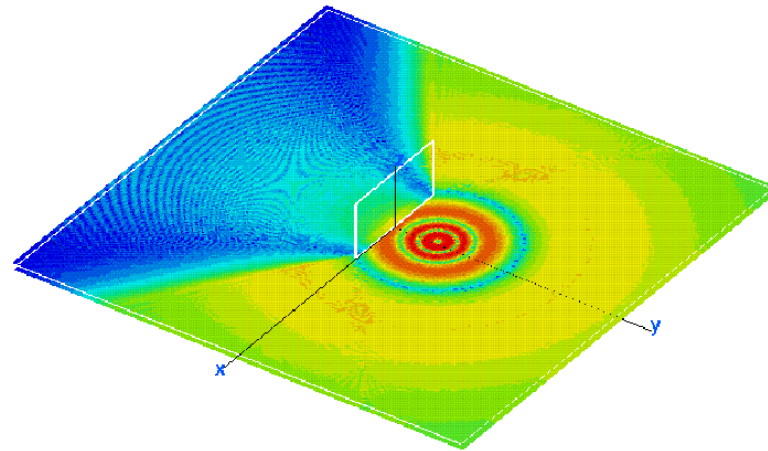


*EC3630 Radiowave Propagation*

# GEOMETRICAL OPTICS AND THE GEOMETRICAL THEORY OF DIFFRACTION

by Professor David Jenn

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(version 1.2)

# Geometrical Optics (1)

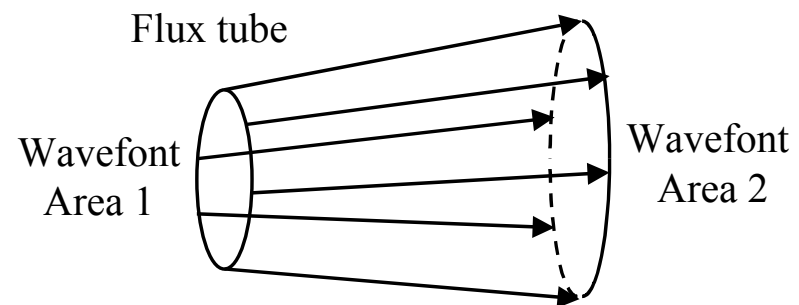
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Geometrical optics (GO) refers to the simple ray tracing techniques that have been used for centuries at optical frequencies. The basic postulates of GO are:

1. Wavefronts are locally plane and waves are TEM
2. The wave direction is specified by the normal to the equiphase planes (“rays”)
3. Rays travel in straight lines in a homogeneous medium
4. Polarization is constant along a ray in an isotropic medium
5. Power in a flux tube (“bundle of rays”) is conserved

$$\iint_{\text{Area 1}} \vec{W} \cdot d\vec{s} = \iint_{\text{Area 2}} \vec{W} \cdot d\vec{s}$$

6. Reflection and refraction obey Snell’s law



7. The reflected field is linearly related to the incident field at the reflection point by a reflection coefficient

# Geometrical Optics (2)

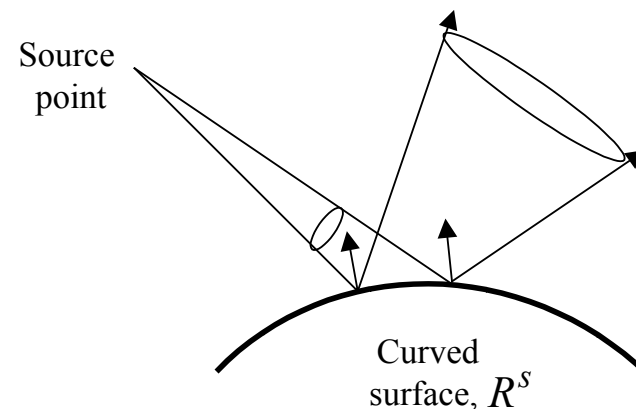
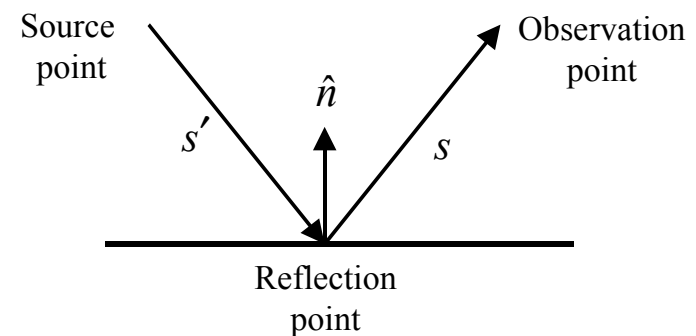
We have already used GO for the simple case of a plane wave reflected from an infinite flat boundary between two dielectrics

For example, for perpendicular polarization:

$$E_{r\perp}(s) = \Gamma_{\perp} E_{i\perp}(s') e^{-jks}$$

where  $s'$  is the distance from the source to the reflection point and  $s$  the distance from the reflection point to the observation point. This has the general form of postulate 7.

The curvature of the reflected wavefront determines how the power spreads as a function of distance and direction. It depends on the curvature of both the incident wavefront,  $R^i$ , and reflecting surface,  $R^s$ .



# Geometrical Optics (3)

A doubly curved surface (or wavefront) is defined by two principal radii of curvature in two orthogonal planes:  $R_1^s, R_2^s$  for a surface or  $R_1^i, R_2^i$  for the incident wavefront. The reflected wavefront curvature ( $R_1^r, R_2^r$ ) can be computed by first finding the focal lengths in the principal planes. When the principal planes of the incident wavefront and surface can be aligned, then

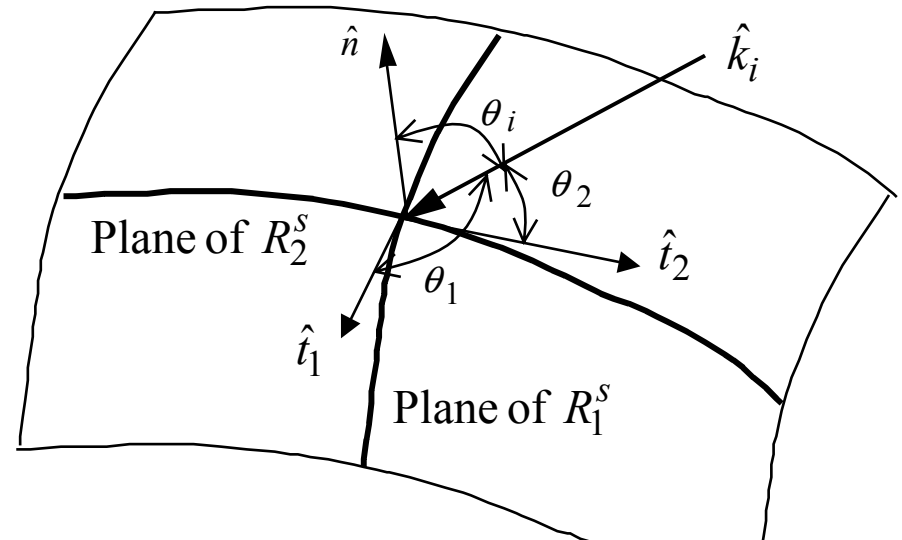
$$\frac{1}{f_{1,2}} = \frac{1}{\cos \theta_i} \left[ \frac{\sin^2 \theta_2}{R_1^s} + \frac{\sin^2 \theta_1}{R_2^s} \right] \pm \left[ \frac{1}{\cos^2 \theta_i} \left( \frac{\sin^2 \theta_2}{R_1^s} + \frac{\sin^2 \theta_1}{R_2^s} \right)^2 - \frac{4}{R_1^s R_2^s} \right]^{1/2}$$

where

$$\frac{1}{R_{1,2}^r} = \frac{1}{2} \left[ \frac{1}{R_1^i} + \frac{1}{R_2^i} \right] + \frac{1}{f_{1,2}}$$

$\hat{t}_1, \hat{t}_2$  = unit vectors tangent to the surface in the two principal planes

$\hat{n}$  = surface normal at the reflection point



# Geometrical Optics (4)

For an arbitrary angle of incidence and polarization, the field is decomposed into parallel and perpendicular components. The reflected field can be cast in matrix form as:

$$\begin{bmatrix} E_{r\perp}(s) \\ E_{r\parallel}(s) \end{bmatrix} = \begin{bmatrix} \Gamma_{\perp\perp} & \Gamma_{\perp\parallel} \\ \Gamma_{\parallel\perp} & \Gamma_{\parallel\parallel} \end{bmatrix} \begin{bmatrix} E_{i\perp}(s') \\ E_{i\parallel}(s') \end{bmatrix} \underbrace{\sqrt{\frac{R_1' R_2'}{(R_1' + s)(R_2' + s)}}}_{A(s), \text{ Spreading Factor}} e^{-jks} e^{j\phi_c}$$

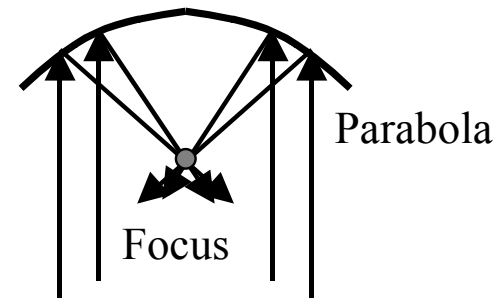
where:

$\phi_c$  = phase change when the path traverses a caustic (a point at which the cross section of the flux tube is zero)

$\Gamma_{pq}$  = reflection coefficient for  $p$  polarized reflected wave,  $q$  polarized incident wave

Disadvantages of GO:

1. does not predict the field in shadows
2. cannot handle flat or singly curved surfaces  
( $R_1^s$  or  $R_2^s = \infty$ )



Example of a caustic: the focus of a parabola. All reflected rays pass through the focus. The cross section of a tube of reflected rays is zero

# Geometrical Optics (5)

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Example: A plane wave is normally incident on a conducting doubly curved surface.

Normal incidence  $\theta_i = 0^\circ \rightarrow \cos \theta_i = 1$  and  $\theta_1 = \theta_2 = 90^\circ \rightarrow \sin \theta_1 = \sin \theta_2 = 1$

$$\frac{1}{f_{1,2}} = \frac{1}{1} \left[ \frac{1}{R_1^s} + \frac{1}{R_2^s} \right] \pm \left[ \left( \frac{1}{R_1^s} + \frac{1}{R_2^s} \right)^2 - \frac{4}{R_1^s R_2^s} \right]^{1/2}$$

If the surface is a sphere of radius  $a$ , then  $R_1^s = R_2^s = a$  and  $f_1 = f_2 = a/2$ . For a plane wave  $R_1^i = R_2^i = \infty$  so that

$$\frac{1}{R_{1,2}^r} = \frac{1}{2} \left[ \frac{1}{R_1^i} + \frac{1}{R_2^i} \right] + \frac{1}{f_{1,2}} = \frac{2}{a} \rightarrow A(s) = \sqrt{\frac{a^2}{s^2}} = \frac{a}{s}$$

where in the denominator it was assumed that  $R_1^r, R_2^r \ll s$ . For a PEC,  $\Gamma = -1$ , and the reflected field is

$$E_r = -E_i a \left( \frac{e^{-jks}}{s} \right)$$

i.e., a spherical wave.

# Geometrical Theory of Diffraction (1)

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The geometrical theory of diffraction (GTD) was devised to eliminate many of the problems associated with GO. The strongest diffracted fields arise from edges, but ones of lesser strength originate from point discontinuities (tips and corners). The total field at an observation point  $P$  is decomposed into GO and diffracted components

$$\vec{E}_r(P) = \vec{E}_{GO}(P) + \vec{E}_{GTD}(P)$$

The behavior of the diffracted field is based on the following postulates of GTD:

1. Wavefronts are locally plane and waves are TEM.
2. Diffracted rays emerge radially from an edge.
3. Rays travel in straight lines in a homogeneous medium
4. Polarization is constant along a ray in an isotropic medium
5. The diffracted field strength is inversely proportional to the cross sectional area of the flux tube
6. The diffracted field is linearly related to the incident field at the diffraction point by a diffraction coefficient

# Geometrical Theory of Diffraction (2)

Define a local edge fixed coordinate system: the  $z$  axis is along the edge; the  $x$  axis lies on the face and points inward.

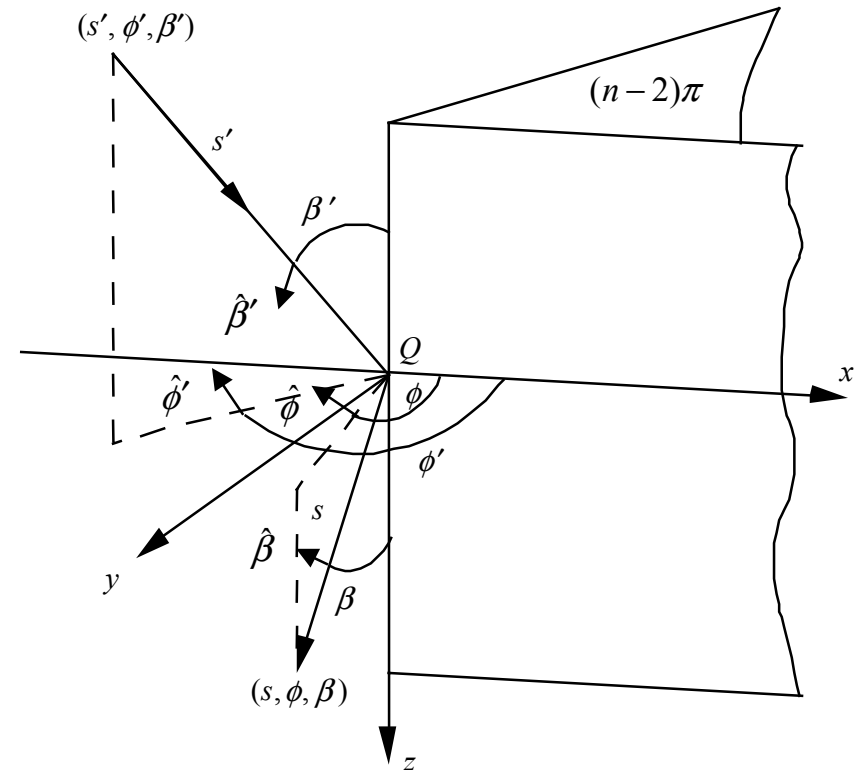
The internal wedge angle is  $(2 - n)\pi$ , where  $n$  is not necessarily an integer. A knife edge is the case of  $n = 2$ .

Primed quantities are associated with the source point; unprimed quantities with the observation point. Variable unit vectors are tangent in the direction of increase (like spherical unit vectors)

Diffracted rays lie on a cone of half angle  $\beta = \beta'$  (the *Keller cone*)

The matrix form of the diffracted field is

$$\begin{bmatrix} E_{d\beta}(s) \\ E_{d\phi}(s) \end{bmatrix} = \begin{bmatrix} D_s & 0 \\ 0 & -D_h \end{bmatrix} \begin{bmatrix} E_{i\beta'}(s') \\ E_{i\phi'}(s') \end{bmatrix} A(s, s') e^{-jks}$$



# Geometrical Theory of Diffraction (3)

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The diffraction coefficients,  $D_s$  and  $D_h$ <sup>1</sup>, are determined by an appropriate “canonical” problem (i.e., a fundamental related problem whose solution is known).

The scattered field from an infinite wedge was solved by Sommerfeld. The infinitely long edge can represent a finite length edge if the diffraction point is not near the end.

The diffraction coefficient can be “backed out” of Sommerfeld’s exact solution because we know the GO field and the form of the GTD field from the postulates

$$\underbrace{\vec{E}_r(P)}_{\substack{\text{known} \\ \text{from} \\ \text{Sommerfeld}}} = \underbrace{\vec{E}_{GO}(P)}_{\substack{\text{can be} \\ \text{found}}} + D \underbrace{\vec{E}_i(Q)A(s,s')e^{-jks}}_{\text{known}}$$

The basic expressions for the diffraction coefficients of a knife edge are simple (they contain only trig functions), however they have singularities at the shadow and reflection boundaries.

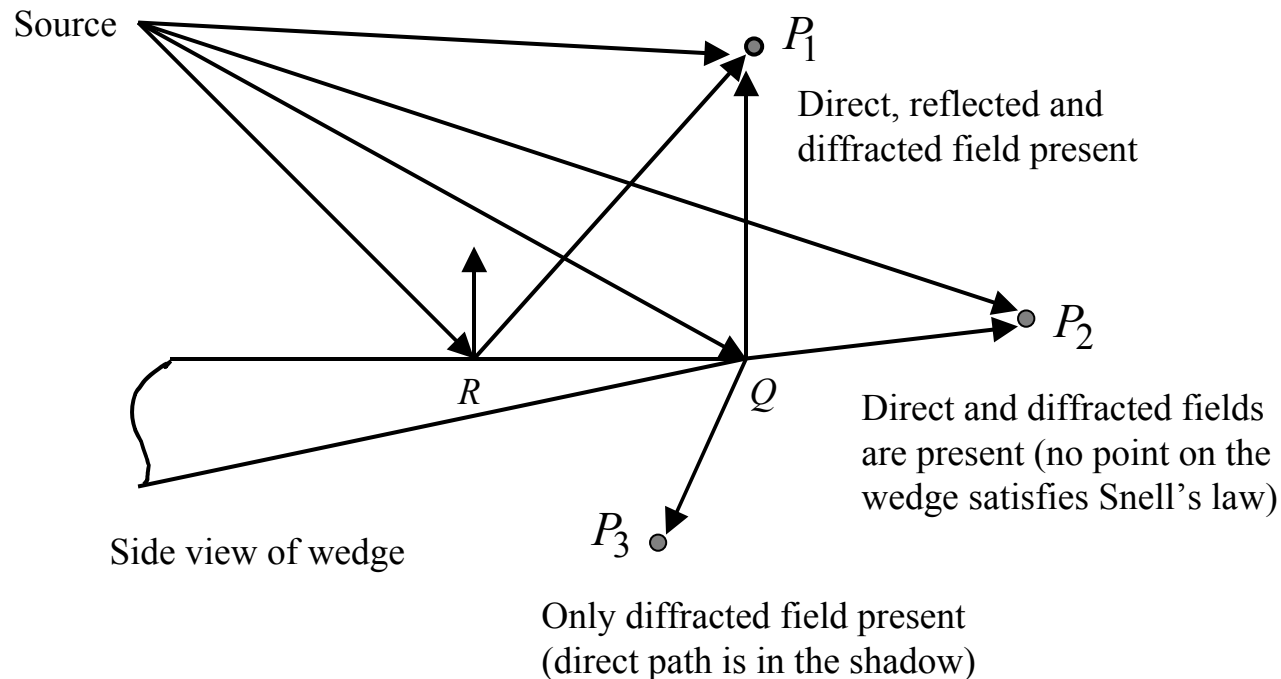
More complicated expressions have been derived that are well behaved everywhere. The most common is the uniform theory of diffraction (UTD). (See Stutzman and Thiele.)

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<sup>1</sup> This notation is borrowed from optics:  $s$  = soft or parallel polarization;  $h$  = hard or perpendicular polarization

# Geometrical Theory of Diffraction (4)

Example: scattering from a wedge for three observation points



GO and GTD rays can “mix” (reflected rays can subsequently be diffracted, etc.) to obtain:

- reflected – reflected (multiple reflection)
- reflected – diffracted and diffracted – reflected
- diffracted – diffracted (multiple diffraction)

An accurate (converged) solution must include all significant contributions.

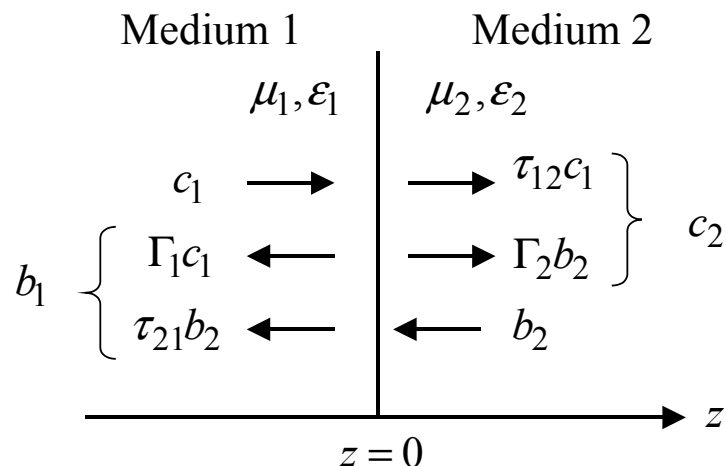
# Wave Matrices For Layered Media (1)

For multilayered (stratified) media, a matrix formulation can be used to determine the net transmitted and reflected fields. The figure below shows incident and reflected waves at the boundary between two media. The positive  $z$  traveling waves are denoted  $c$  and the negative  $z$  traveling waves  $b$ . We allow for waves incident from both sides simultaneously.

Thus,

$$\begin{aligned} b_1 &= \Gamma_1 c_1 + \tau_{21} b_2 \\ c_2 &= \Gamma_2 b_2 + \tau_{12} c_1 \end{aligned}$$

where  $\Gamma$  and  $\tau$  are the appropriate Fresnel reflection and transmission coefficients.



Rearranging the two equations:

$$\begin{aligned} b_1 &= \left( \tau_{21} - \frac{\Gamma_1 \Gamma_2}{\tau_{12}} \right) b_2 + \frac{\Gamma_1}{\tau_{12}} c_2 \\ c_1 &= \frac{c_2}{\tau_{12}} - \frac{\Gamma_2 b_2}{\tau_{12}} \end{aligned}$$

which can be written in matrix form:

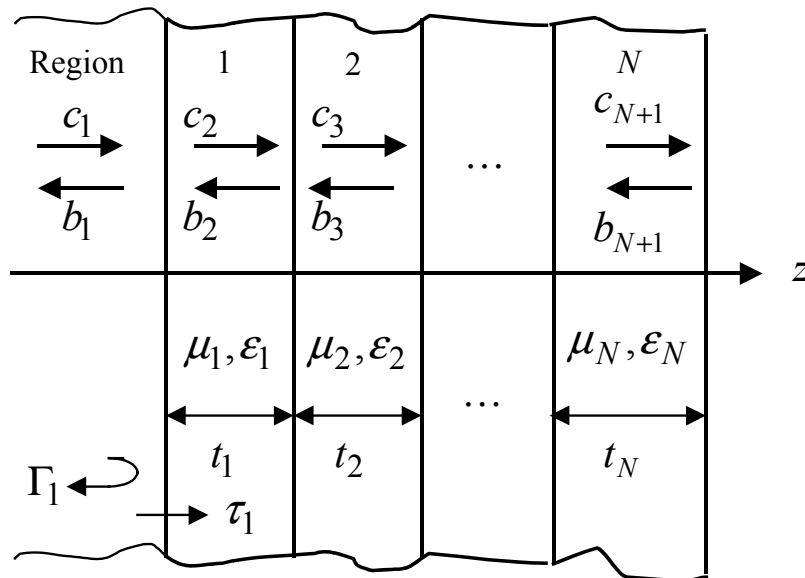
$$\begin{bmatrix} c_1 \\ b_1 \end{bmatrix} = \frac{1}{\tau_{12}} \begin{bmatrix} 1 & -\Gamma_2 \\ \Gamma_1 & \tau_{12} \tau_{21} - \Gamma_1 \Gamma_2 \end{bmatrix} \begin{bmatrix} c_2 \\ b_2 \end{bmatrix}$$

This is called the wave transmission matrix. It relates the forward and backward propagating waves on the two sides of the boundary.

# Wave Matrices For Layered Media (2)

As defined,  $c$  and  $b$  are the waves incident on the boundary,  $z = 0$ . At some other location,  $z = z_1$  the forward traveling wave becomes  $c_1 e^{-j\beta z_1}$  and the backward wave becomes  $b_1 e^{j\beta z_1}$ . For a plane wave incident from free space onto  $N$  layers of different material  $(\mu_n, \epsilon_n)$  and thickness  $(t_n)$ , the wave matrices can be cascaded

$$\begin{bmatrix} c_1 \\ b_1 \end{bmatrix} = \prod_{n=1}^N \frac{1}{\tau_n} \begin{bmatrix} e^{j\Phi_n} & \Gamma_n e^{-j\Phi_n} \\ \Gamma_n e^{j\Phi_n} & e^{-j\Phi_n} \end{bmatrix} \begin{bmatrix} c_{N+1} \\ b_{N+1} \end{bmatrix} \equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} c_{N+1} \\ b_{N+1} \end{bmatrix}$$



where, for normal incidence,  $\Phi_n = \beta_n t_n$  is the electrical length of layer  $n$ . If the last layer extends to  $z \rightarrow \infty$  then  $b_{N+1} = 0$  and we can use  $\Phi_N = 0$ .

The overall transmission coefficient of the layers (i.e., the transmission into layer  $N$  when  $b_{N+1} = 0$ ) is  $c_{N+1} / c_1 = 1 / A_{11}$ . The overall reflection coefficient is  $b_1 / c_1 = A_{21} / A_{11}$ .

# Wave Matrices For Layered Media (3)

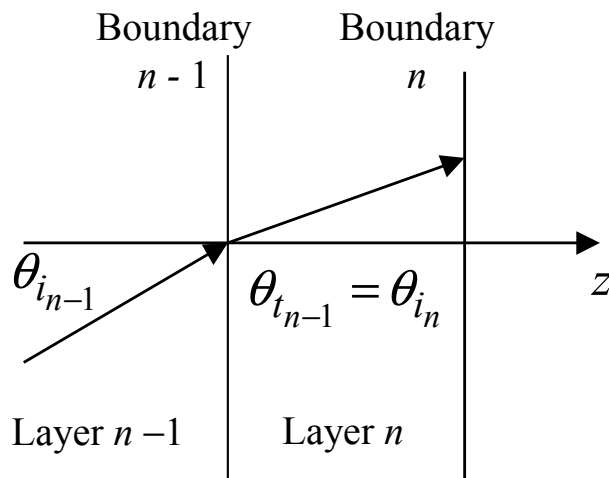
If the incidence angle in region 1 is not normal, then  $\Phi_n$  must be determined by taking into account the refraction in all of the previous  $n - 1$  layers. The transmission angle for layer  $n$  becomes the incidence angle for layer  $n + 1$  and they are related by Snell's law

$$\beta_1 \sin \theta_{i_1} = \beta_2 \sin \theta_{t_2} = \beta_2 \sin \theta_{i_2}.$$

Therefore,

$$\beta_o \sin \theta_i = \beta_1 \sin \theta_{i_1} = \beta_2 \sin \theta_{i_2} = \dots = \beta_{N-1} \sin \theta_{i_{N-1}}.$$

Oblique incidence can be handled by modifying the transmission and reflection formulas as listed below, where  $\theta_i$  is the incidence angle at the first boundary:



$$\Gamma_n = \frac{Z_n - Z_{n-1}}{Z_n + Z_{n-1}} \quad \tau_n = 1 + \Gamma_n$$

$$Z_n = \frac{(\epsilon_{r_n} - \sin^2 \theta_i)^{1/2}}{\epsilon_{r_n} \cos \theta_i}$$

$$\Phi_n = \beta_o t_n (\epsilon_{r_n} - \sin^2 \theta_i)^{1/2}$$

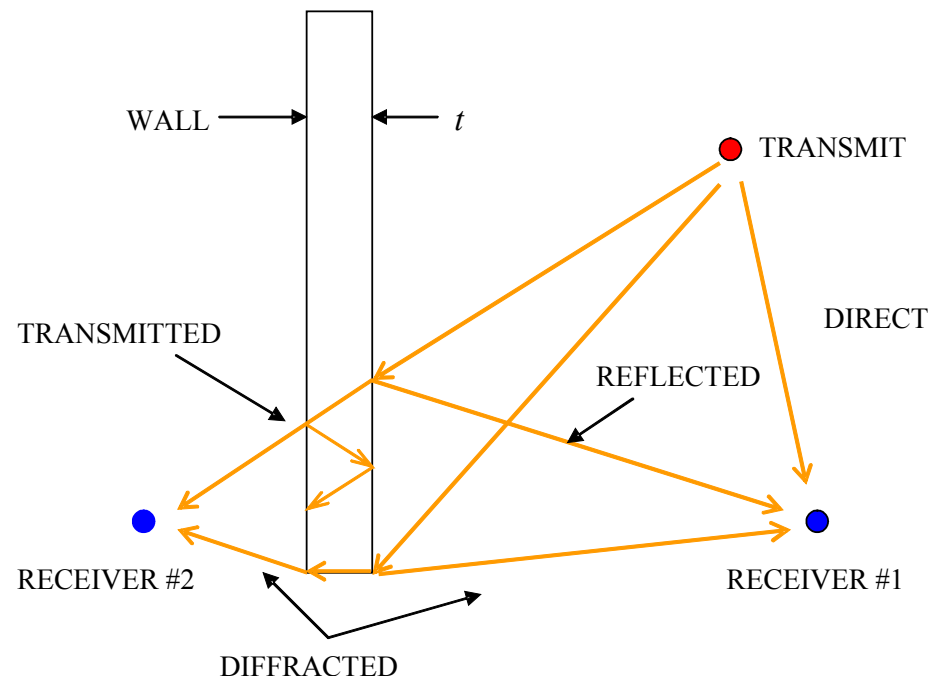
# Indoor and Urban Propagation Modeling (1)

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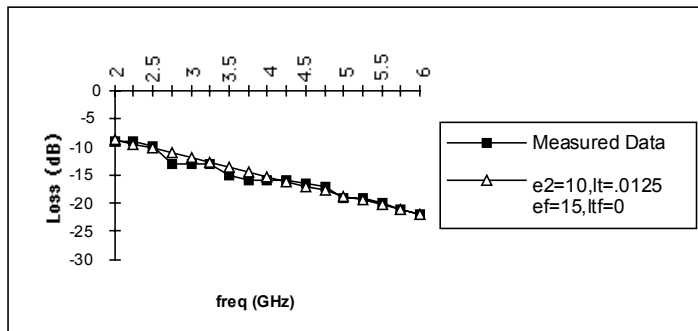
- An important application for which the use of GO and GTD are well suited, is the modeling of propagation for wireless systems in buildings and urban environments.
- Applications include
  1. wireless local area networks (WLANs)
  2. mobile communications systems
  3. cellular phones
  4. command, control, and data links for UAVs flying in cities
  5. high power microwaves (both attack and protect)
  6. GPS performance in urban environments
- These systems generally operate at frequencies above 900 MHz (some European wireless systems operate at frequencies as low as 400 MHz). High frequency methods are applicable in this range.
- Computational electromagnetics (CEM) codes are used to predict the propagation of electromagnetic (EM) waves indoors and in urban environments.

# Indoor and Urban Propagation Modeling (2)

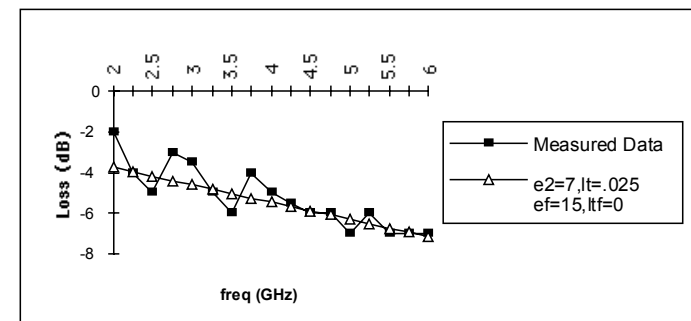
- Scattering properties of common building materials are determined by their permittivity, permeability, and conductivity
- Sources of loss (attenuation) inside of walls:
  1. absorption (energy dissipated inside of material)
  2. cancellation by reflection
- Propagation and interaction with materials is decomposed into
  1. transmission
  2. reflection
  3. diffraction from discontinuities
- Usually we do not know exactly what is inside of a wall (plumbing, wiring, duct work, insulation, etc.)



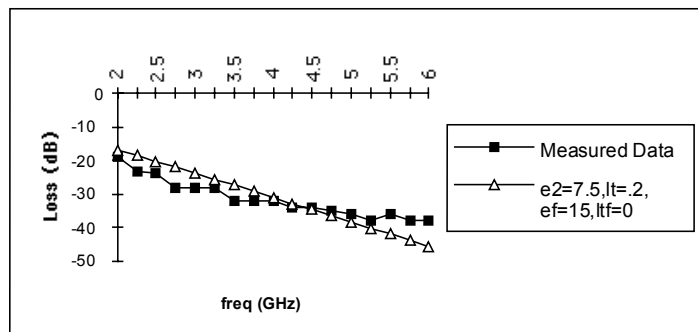
# Propagation Loss Through Walls



Loss through a 10 inch concrete wall



Loss through a 1.75 inch wood doors



Loss through 1.75 inch metal doors



Measurement of propagation through building walls

# Propagation Loss Through Windows

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Closed blinds



Insertion loss about 10 dB

Window tinting film



Insertion loss about 20 dB

# WLAN Antennas (Omnidirectional)

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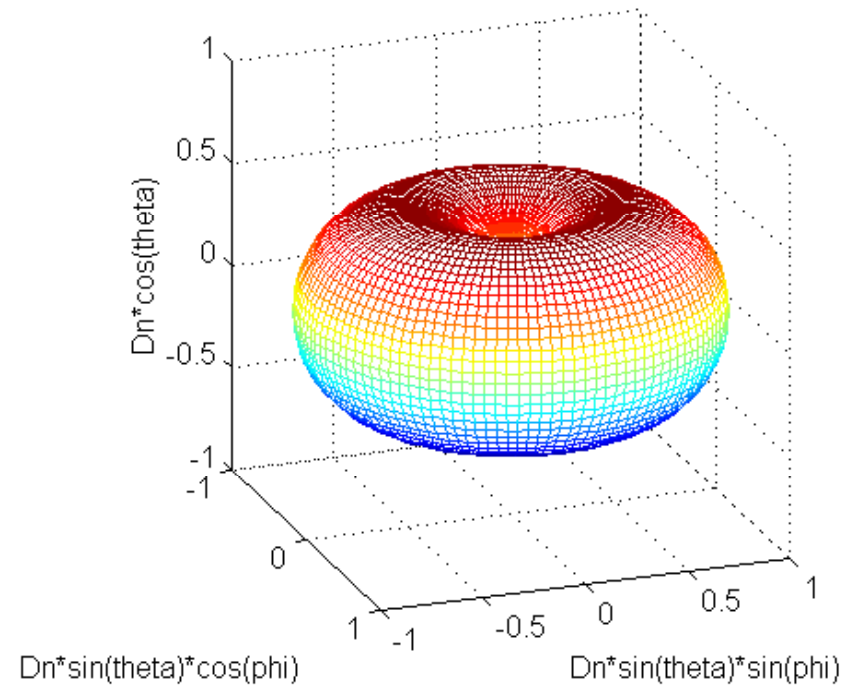
- Ceiling mount



- Desktop mount diversity antennas



- Spatial radiation distribution of a vertical dipole antenna



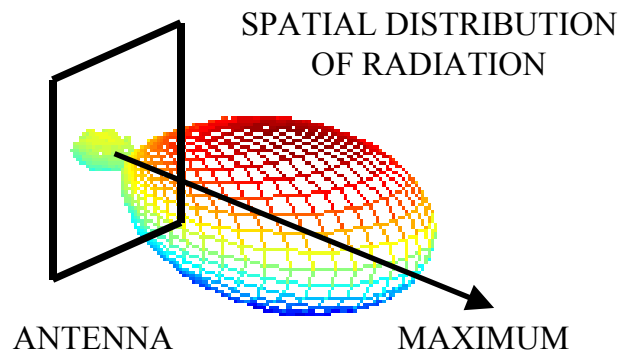
# WLAN Directional Antennas

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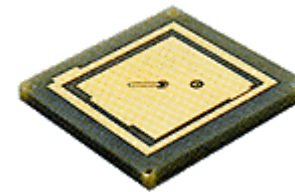
Microstrip patch antenna  
(typically mounted on a wall)



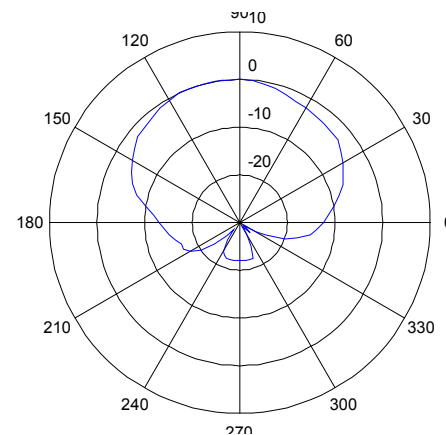
Radiation pattern (power measured as  
a function of angle at constant radius)



Radome cover removed



Typical antenna pattern  
(top view)



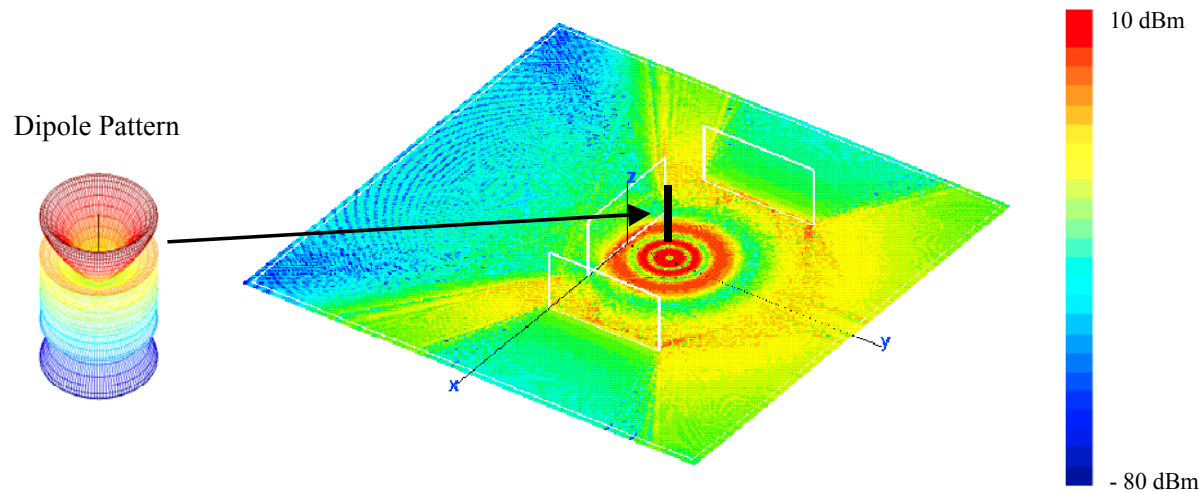
# *Urbana* Wireless Toolset

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- Components
  1. *XCell*: geometry builder and visualizer; antenna placement; observation point definition
  2. *Cifer*: utilities, translators, geometry manipulation
  3. *Urbana*: electromagnetic solvers
- Features:
  1. Interfaces with computer aided design (CAD) software
  2. Reflections by geometrical optics (GO) or “shooting and bouncing rays” (SBR)
  3. Diffraction by geometrical theory of diffraction (GTD) or physical theory of diffraction (PTD)
  4. Surface and edge curvature can be modeled
  5. Complex materials (dielectrics, conductors, magnetic material)
- The SGI version is not “user friendly”

# Simple Three Wall Example

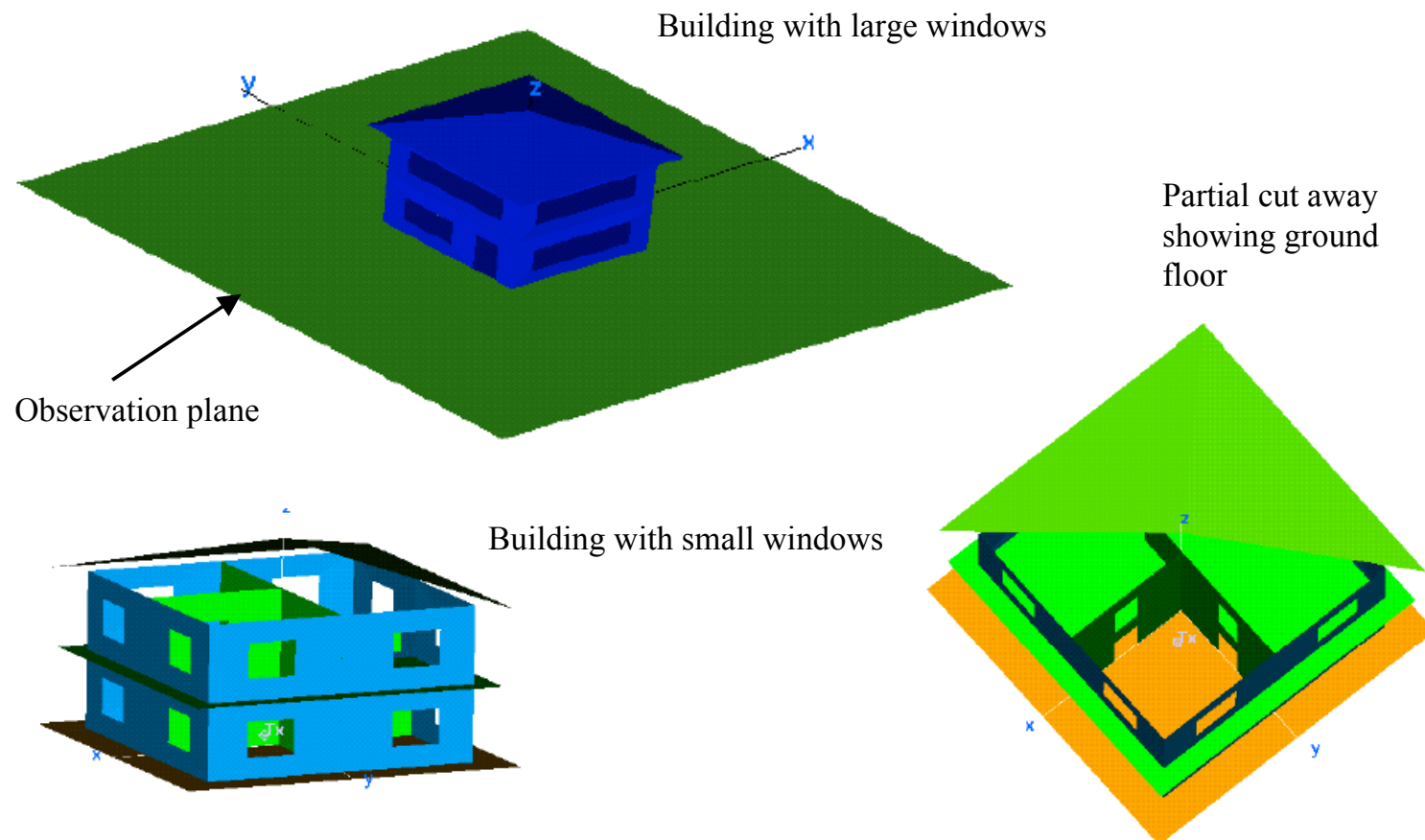
- Dipole behind walls (edges shown white)
- 1 watt transmit power; plastic walls are 25m by 50m; 8 wavelength dipole antenna



- Propagation features can be identified:
  - > dipole radiation rings (red)
  - > multipath from ground and wall surfaces (red speckle)
  - > propagation through gaps
  - > “shadows” behind walls (shadow boundaries from wall edges)
  - > diffraction from wall edges (blue arcs)

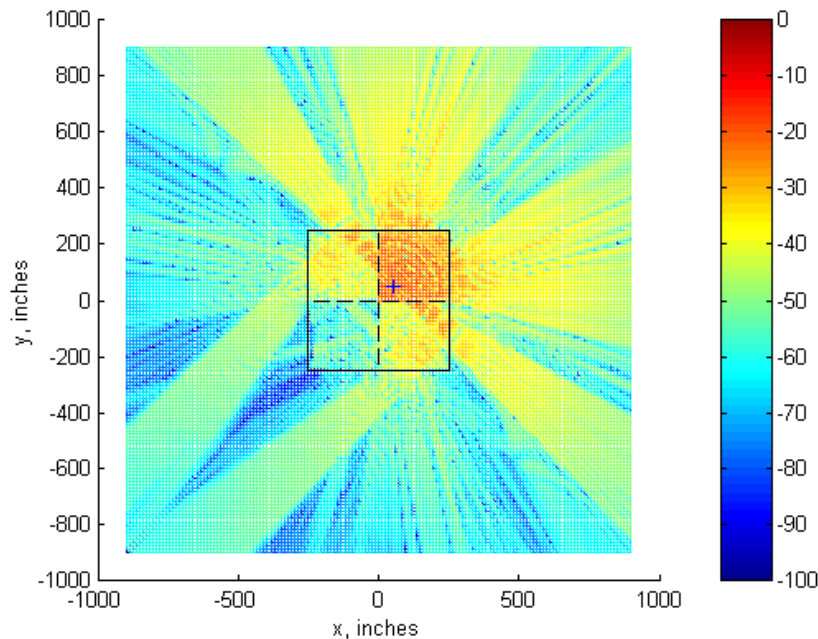
# Two Story Building

Two story building: 40 feet on a side. Observation plane is 150 feet on a side.

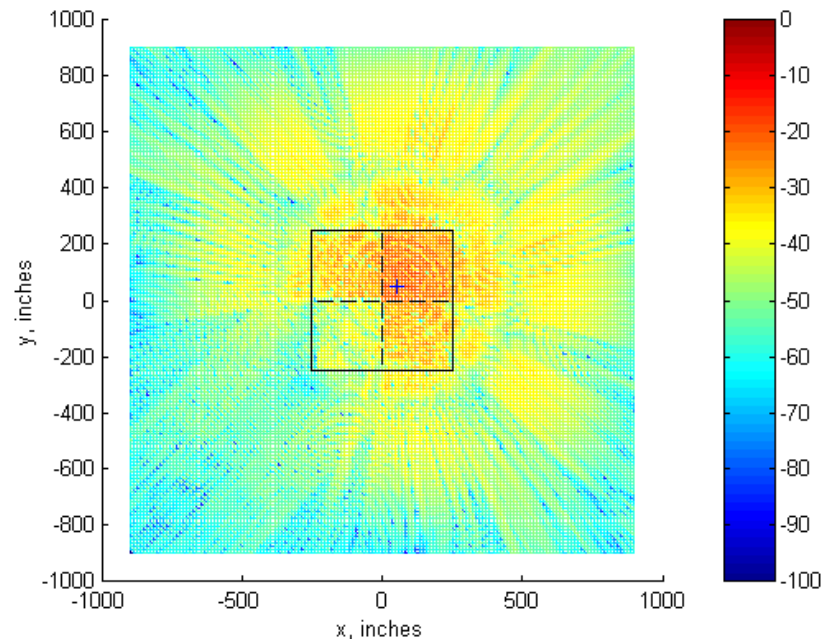


# Wall Materials

## Metal composite walls



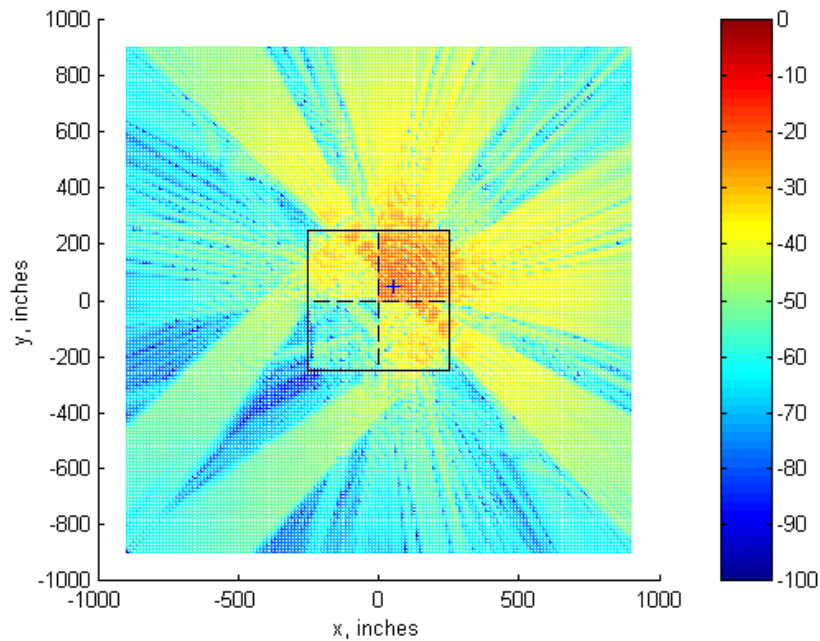
## Wood walls



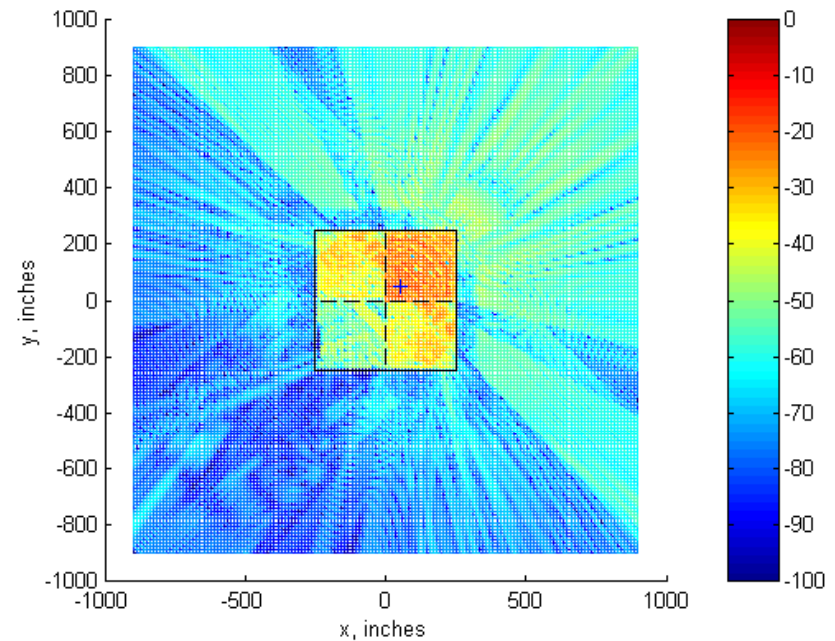
- Propagation through windows dominates for metal buildings
- Many transmissions and reflections diffuse the signal for the wood building
- Receive antenna is 5 feet above ground

# Window Materials

Standard glass windows



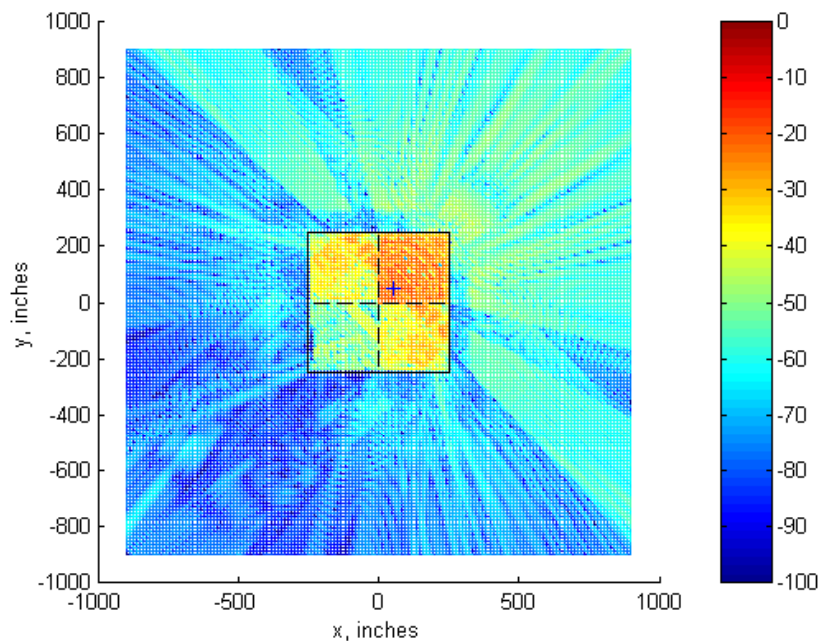
Tinted windows



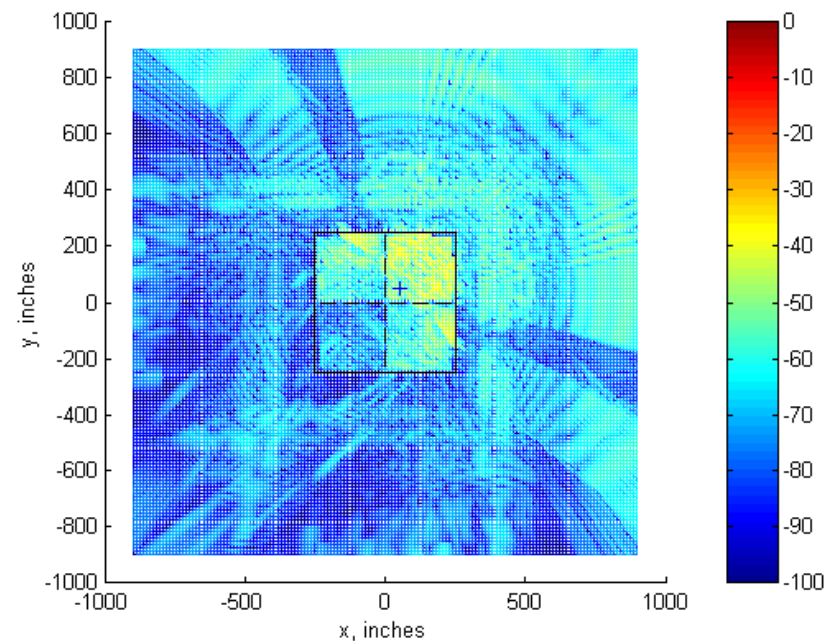
- There is a direct line of sight above the window sill from inside the building
- Receive antenna is 5 feet above ground

# Antenna Location

Access point on 1<sup>st</sup> floor



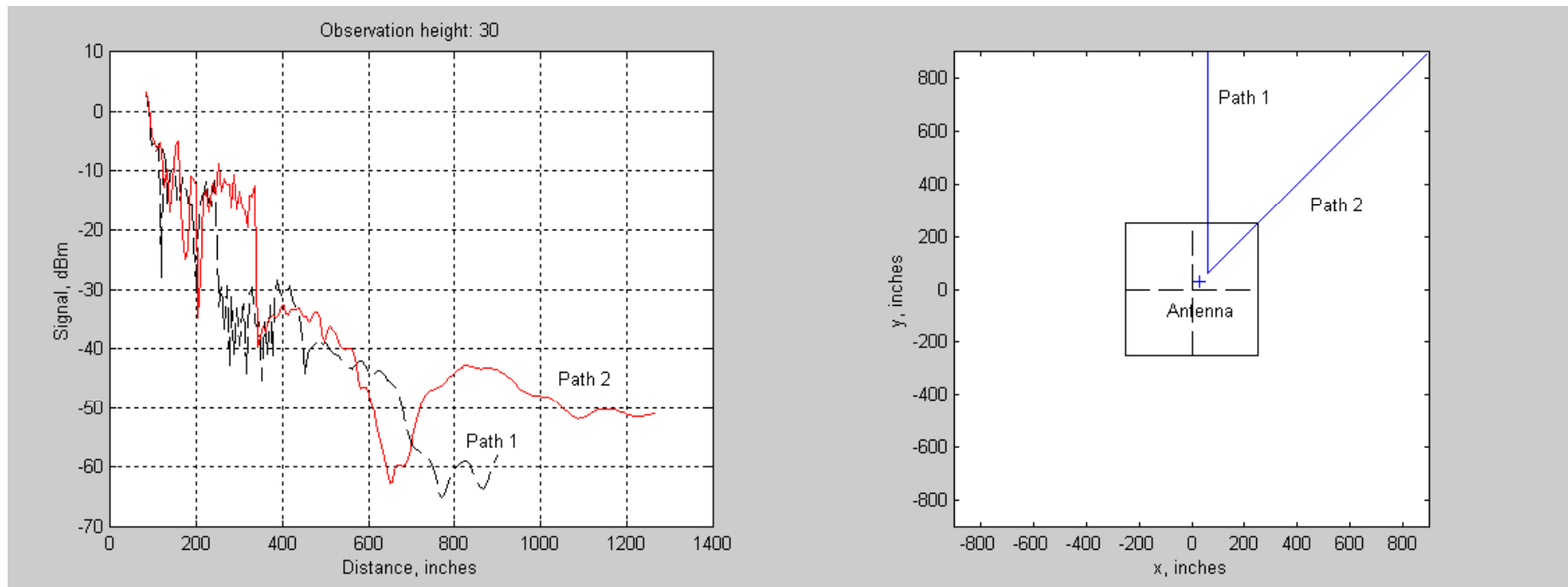
Access point on 2<sup>nd</sup> floor



- Detection is reduced by moving the access point to second floor
- Results in reduced signal levels inside on first floor
- Receive antenna is 5 feet above ground

# Field Along a Line Path Through a Wall

- Metal composite walls, standard glass
- 10 to 20 dB drop through the wall

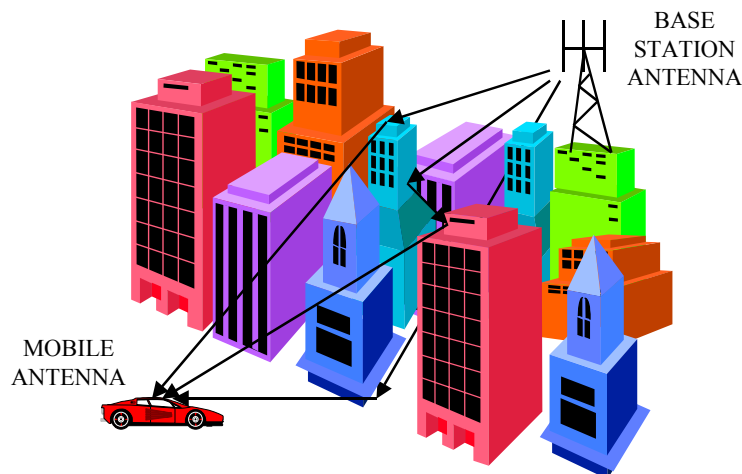


# Urban Propagation (1)

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Urban propagation is a unique and relatively new area of study. It is important in the design of cellular and mobile communication systems. A complete theoretical treatment of propagation in an urban environment is practically intractable. Many combinations of propagation mechanisms are possible, each with different paths. The details of the environment change from city to city and from block to block within a city. Statistical models are very effective in predicting propagation in this situation.

In an urban or suburban environment there is rarely a direct path between the transmitting and receiving antennas. However there usually are multiple reflection and diffraction paths between a transmitter and receiver.



- Reflections from objects close to the mobile antenna will cause multiple signals to add and cancel as the mobile unit moves. Almost complete cancellation can occur resulting in “deep fades.” These small-scale (on the order of tens of wavelengths) variations in the signal are predicted by Rayleigh statistics.

# Urban Propagation (2)

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- On a larger scale (hundreds to thousands of wavelengths) the signal behavior, when measured in dB, has been found to be normally distributed (hence referred to a lognormal distribution). The genesis of the lognormal variation is the multiplicative nature of shadowing and diffraction of signals along rooftops and undulating terrain.
- The Hata model is used most often for predicting path loss in various types of urban conditions. It is a set of empirically derived formulas that include correction factors for antenna heights and terrain.

Path loss is the  $1/r^2$  spreading loss in signal between two isotropic antennas. From the Friis equation, with  $G_t = G_r = 4\pi A_e / \lambda^2 = 1$

$$L_s = \frac{P_r}{P_t} = \frac{(1)(1)\lambda^2}{(4\pi r)^2} = \left(\frac{1}{2kr}\right)^2$$

Note that path loss is not a true loss of energy as in the case of attenuation. Path loss as defined here will occur even if the medium between the antennas is lossless. It arises because the transmitted signal propagates as a spherical wave and hence power is flowing in directions other than towards the receiver.

# Urban Propagation (3)

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Hata model parameters<sup>2</sup>:  $d$  = transmit/receive distance ( $1 \leq d \leq 20$  km)  
 $f$  = frequency in MHz ( $100 \leq f \leq 1500$  MHz)  
 $h_b$  = base antenna height ( $30 \leq h_b \leq 200$  m)  
 $h_m$  = mobile antenna height ( $1 \leq h_m \leq 10$  m)

The median path loss is

$$L_{\text{med}} = 69.55 + 26.16 \log(f) - 13.82 \log(h_b) + [44.9 - 6.55 \log(h_b)] \log(d) + a(h_m)$$

In a medium city:  $a(h_m) = [0.7 - 1.1 \log(f)] h_m + 1.56 \log(f) - 0.8$

In a large city:  $a(h_m) = \begin{cases} 1.1 - 8.29 \log^2(1.54 h_m), & f \leq 200 \text{ MHz} \\ 4.97 - 3.2 \log^2(11.75 h_m), & f \geq 400 \text{ MHz} \end{cases}$

Correction factors:  $L_{\text{cor}} = \begin{cases} -2 \log^2(f/28) - 5.4, & \text{suburban areas} \\ -4.78 \log^2(f) + 18.33 \log(f) - 40.94, & \text{open areas} \end{cases}$

The total path loss is:  $L_s = L_{\text{med}} - L_{\text{cor}}$

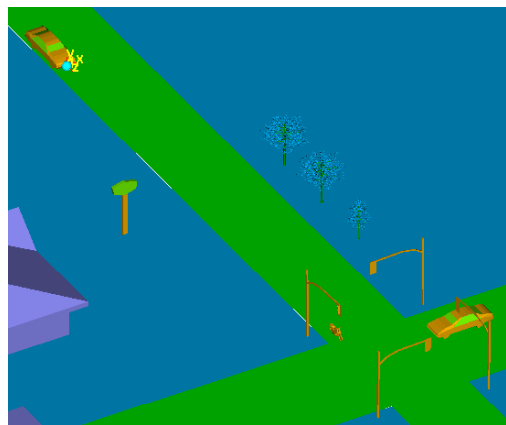
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<sup>2</sup> Note: Modified formulas have been derived to extend the range of all parameters.

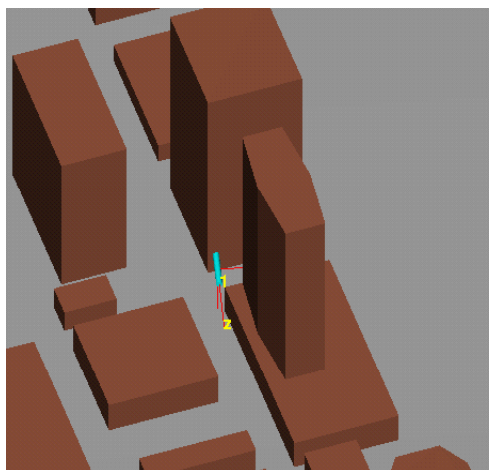
# Urban Propagation Models

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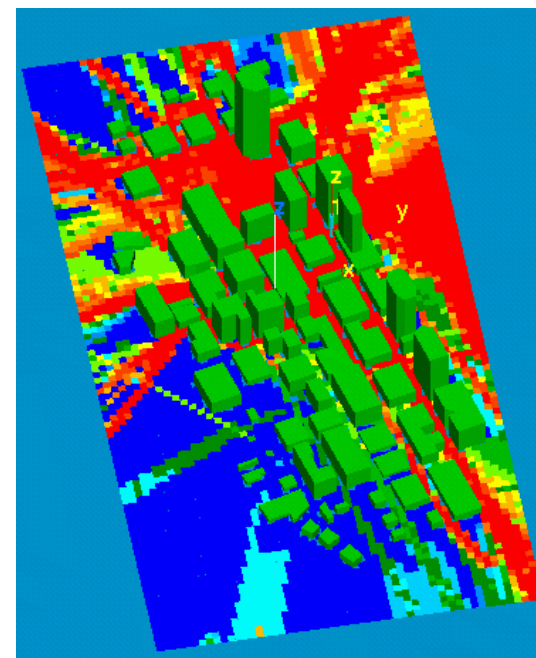
- Examples of urban propagation models and field distributions



Suburban model with  
buildings and roads



Transmitter antenna in  
urban location

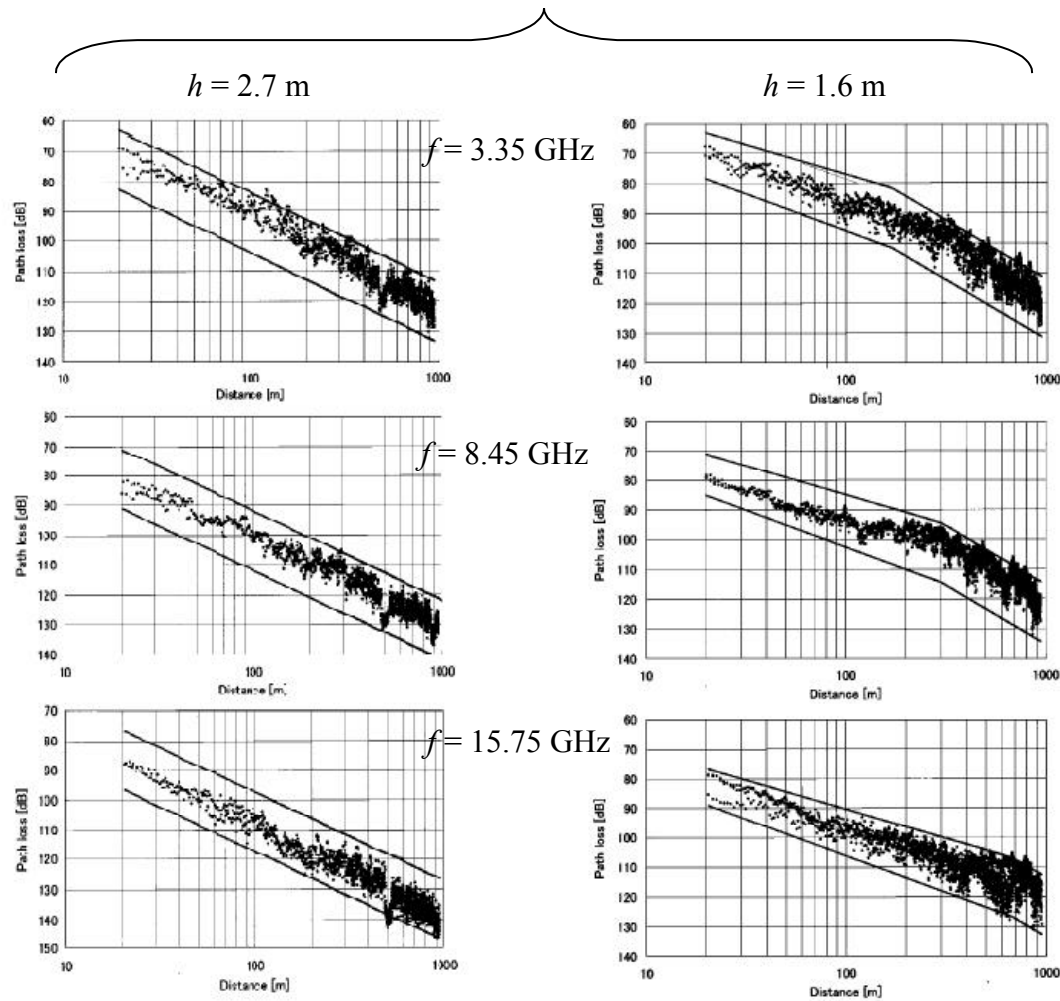


Signal strength distribution

From SAIC's *Urbana Wireless Toolkit*

# Measured Data

Two different antenna heights



Measured data in  
an urban  
environment

Three different  
frequencies

From Masui, "Microwave Path Loss  
Modeling in Urban LOS Environ-  
ments," *IEEE Journ. on Selected Areas  
in Comms.*, Vol 20, No. 6, Aug. 2002.